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MATRIX GAME WITH THE PREFERENCE CHANGING IN TIME

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Abstract

In this paper matrix game is defined, which elements are the functions of one argument differentiated for any kind of row in the [0, 1] interval. The following cases are discussed: 1). The functions are the polynomials not more of m-1 degree; 2). The functions are more common and have every kind of row uninterrupted derived in any positive interval $[0, t_0] \subset [0,1]$. For these matrix games the solutions (or saddle points) are defined in the positive interval in the pure and mixed strategies. The questions of their existence are taken from the existence the solutions in the lexicographic matrix games m-measuring vectorial payoffs in the first case, during the unlimited measuring vectorial payoffs in the second case.

Keywords: matrix game, lexicographic game, solution.

1. INTRODUCTION

Let us set a problem: two firms (two players) 1 and 2 try to sell their production on one and the same market. 1 can produce *p* type, but 2 - *q* type of production, their purpose is the same. Hence, $S_1 = \{1, \ldots, p\}$ and $S_2 = \{1, \ldots, q\}$ are the sets of the player's strategies. Suppose, it is stated by prediction, that in the (*i, j*) situation players' utilities (payoffs) are depend on $t \in [0,1]$ time and respectively it is given by $f_{ij}^1(t)$ and $f_{ij}^2(t)$ functions. Hence $f_{ij}^k(t), k = 1, 2$ is k player's payoff in t moment and in the (*i, j*) situation. Let's suppose that the aim of each firm is choosing such strategy (type of production), by which it will get maximal payoff after entering the market at the incipient stage. According to this, we can say that for k firm the situation (*i, j*) is preferable on the (i^1, j^1) situation and we'll write $(i, j) \succ_k (i^1, j^1)$, if exists such $t_0 > 0$, that for any $0 < t \le t_0$ will fullfil the following inequality $f_{ij}^k(t) \ge f_{i^1 j^1}^k(t)$.

Hence, we've got two players' non-cooperative nonantagonistic game with functional elements

$$\Gamma_1 = < \{1,2\}, S_1, S_2, (\mathrm{H}_1, \mathrm{H}_2) = \{ (f_{ij}^1(t), f_{ij}^2(t)) \}, t \in [0,1] >,$$

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where, a player in the process of choosing the strategy considers the change of preference in time. It's obvious that this game is a bimatrix game.

Let us suppose, that in the condition of Γ_1 game

$$f_{ij}^1(t) = - f_{ij}^2(t) \equiv f_{ij}(t),\ \forall\,(i, j) \in S_1 X S_2, \forall t \in [0,1].$$

In this way we'll get a two-person zero-sum game or matrix game with functional elements

$$\Gamma = <\{1,2\}, S_1, S_2, H >= \{\ f_{ij}(t)\ \},\ \ t \in [0,1] >.$$

In accordance with this, we consider that finite non-cooperative games (with finite set of players and with the player's sets of finite strategies) with functional elements must be discussed. Matrix Γ game is discussed in the article, where $f_{ij}(t)$ functions are uninterrupted derived differentiated for any kind of rou on $[0,1]$ interval and its solution in the pure and mixed strategies is defined.

2. THE MAIN PART

Let us discuss pxq-matrix game $\Gamma_{F(t)} \equiv \Gamma_F$ with payoff matrix $F\ (t)$, which elements represent the following functions

$$f_{ij} : [0,1] \to R^1, i = 1,\ldots, p;\ j = 1,\ldots,q \qquad (1)$$ Let us note

$$F(t) = \{\ f_{ij}(t)\}, i = 1,\ldots, p;\ j = 1,\ldots,q \qquad (2)$$

Let's take the following definitions.

Definition 2.1. The situation (i^*,j^*) in matrix game Γ_F with payoff matrix $F(t)$ is called a saddle point (or a solution) if exists such positive number $t_0 \in [0,1]$, that for every $t_0 \in [0,t_0]$ the following inequalities will take place

$$f_{ij^*}(t) \le f_{i^* j^*}(t) \le f_{i^* j}(t),\ \forall i, \forall j. \qquad (3)$$

Definition 2.2. The situation (X^*, Y^*) in mixed strategies in matrix game Γ_F with payoff matrix $F(t)$ is called a saddle point (or a solution) if exists such positive number $t_0 \in [0,t_0]$, that for every $t \in [0,t_0]$ will fulfil the following inequalities

$$XF(t)Y^{*T} \le X^* F(t) Y^{*T} \le X^* F(t) Y^T,\ \forall\ X \in \aleph_1, \forall\ Y \in \aleph_2. \qquad (4)$$

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After the fulfil (3) inequalities we'll say that (*i**,*j**) is the saddle point of game Γ_F on $[0,t_0]$ interval and we'll write (*i**,*j**)∈ *G* $(\Gamma_F[0,t_0])$. Analogically, after the fulful (4) we'll say that (X^*,Y^*) is the saddle point of game Γ_F on $[0,t_0]$ interval and we'll write $(X^*,Y^*)\in$ *G* $(\overline{\Gamma}_F[0,t_0])$.

As in the scalar matrix game (Vorob'ev, 1994) here: (*i**,*j**)∈ *G* $(\Gamma_F[0,t_0])$ if and only if when

$$\max_i \min_j f_{ij}(t) = \min_j \max_i f_{ij}(t) = f_{i^*j^*}(t), \ \forall t \in [0,t_0];$$

$(X^*,Y^*)\in G(\overline{\Gamma}_F[0,t_0])$ if and only if when $F_{i.}(t)Y^{*T} \le X^*F(t)Y^{*T} \le X^*F_{j.}(t), \ \forall i, \forall j, \forall t \in [0,t_0]$,

where $F_{i\cdot}(t)$ note i th row and $F_{j\cdot}(t)-$ j th column of $F(t)$ matrix, and the meaning game Γ_F is equal

$$v(F(t)) = \max_X \min_Y XF(t)Y^T = \min_Y \max_X XF(t)Y^T = X^*F(t)Y^{*T}.$$

It has come out that, in the matrix Γ_F game the existence of solution in the mixed strategies is connected to the existence of the solution in the lexicographic matrix game, that represent lexicographic non-cooperative $\Gamma^L = (\Gamma^1,\ldots,\Gamma^m)$ game's private case (Salukvadze et.al. 2009).

It's known, that (Fishburn, 1972; Podinovski, 1973) in the lexicographic matrix game the solution either in the pure nor in the mixed strategies may not be exist. In the lexicographic finite non-cooperative and matrix $\Gamma^L = (\Gamma^1,\ldots,\Gamma^m)$ games the problem of the existense of the equiilibrium situation has been solved by means of $m-1$ degree scalar (parametrix) affine games (Beltadze, 1980, 1981, 1991). Namely, $m-1$ degree afine game has the following form

$$\Gamma_{(t)} = \Gamma^1 + \sum_{k=2}^{m} (\Gamma^k - \Gamma^{k-1})t^{k-1}, t \in [0,1], \qquad (5)$$

in which the matrix of payoff will be denoted by a corresponding combination. By means of (5) it is shown that the set of the solutions of game Γ^L are given in the following form:

$G(\overline{\Gamma}^L) = \bigcup_{\tau\in(0,1]} \bigcap_{t\in[0,\tau]} G(\overline{\Gamma}_{(t)})$ (6) Let us go back to Γ_F game and discuss the following cases.

1). Γ_F game payoff's $F(t)$ matrix elements defined by (2) are polynomials $f_{ij}(t)=P_{ij}(t)$ not more *m*-1 degree:

$$P_{ij}(t)=C_{ij}^0+C_{ij}^1 t+\cdots C_{ij}^{m-1}t^{m-1}, \quad i=1,\cdots,p; \quad j=1,\cdots q.$$

Let, lexicographic matrix $\Gamma^L=(\Gamma^1,\dots,\Gamma^m)$ game payoff's matrix is

$$\mathrm{H}=(\mathrm{H}^1,\dots,\mathrm{H}^m)=\{(a_{ij}^1,\dots,a_{ij}^m)\},\ i=1,\dots,p;\ j=1,\dots,q.$$

Then we can consider Γ_F game as the affine matrix game $\Gamma_{(t)}$ of $m-1$ degree of the lexicographic game Γ^L with the payoff of $\mathrm{H}=(\mathrm{H}^1,\dots,\mathrm{H}^m)=\{(a_{ij}^1,\dots,a_{ij}^m)\}$ defined by (5).

Theorem 2.1. The matrixes of payoffs' H and $F(t)$ are denoting one valued each other.

Proof. Really, on the (5) basis, using of H the matrix $\Gamma_{(t)}$ affine game payoff's matrix is defined only in this way

$$\mathrm{H}_{(t)}=\mathrm{H}^1+(\mathrm{H}^2-\mathrm{H}^1)t+\dots+(\mathrm{H}^m-\mathrm{H}^{m-1})t^{m-1}\equiv\{a_{ij}(t)\},$$

$$i=1,\dots,p;\ j=1,\dots,q,$$

where $a_{ij}(t)=a_{ij}^1+(a_{ij}^2-a_{ij}^1)t+\dots+(a_{ij}^m-a_{ij}^{m-1})t^{m-1}$. It means that $\mathrm{H}_{(t)}=F(t)=\{P_{ij}(t)\}$.

Now, on the contrary, let $F(t)$ matrix is given:

$$F(t)=\{P_{ij}(t)\}=\{C_{ij}^0+C_{ij}^1 t+\dots+C_{ij}^{m-1}t^{m-1}\}.$$

If we compare $P_{ij}(t)$ and $a_{ij}(t)$ to each other we can write

$$a_{ij}^1=C_{ij}^0, a_{ij}^2-a_{ij}^1=C_{ij}^1,\dots,a_{ij}^m-a_{ij}^{m-1}=C_{ij}^{m-1}$$

for each i and j. From these equilities we can define $a_{ij}=(a_{ij}^1,\dots,a_{ij}^m)$ and we'll get H matrix.

Example 2.1. If

$$F(t)=\begin{pmatrix}2+t-t^2 & 3t+6t^2\\ -3t & 1+4t^2\end{pmatrix}, \quad \text{then } \mathrm{H}=\begin{pmatrix}(2,3,2) & (0,3,9,)\\ (0,-3,-3) & (1,1,5)\end{pmatrix}.$$

Correlation (6) between $G(\overline{\Gamma}^L)$ and $G(\overline{\Gamma}_{(t)})$ the sets is given so:

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$$G\left(\overline{\Gamma}^L\right) = G\left(\Gamma_F[0,t_0]\right). \quad (7)$$

In view of (7) like of the case of scalar matrix games the following theorem is proving.

Theorem 2.2. In order to exist the solution in the pure strategies in the matrix Γ_F game (or lexicographic martix game Γ^L) it is necessary and sufficient, that on any positive interval $t \in [0,t_0]$ have to fulfil the equality

$$\underline{v(t)} = \max_i \min_j P_{ij}(t) = \min_j \max_i P_{ij}(t) = \overline{v(t)}.$$

Let us note that in the lexicographic matrix games (as well as in the noncoalition lexicographic games) in order to establish and carry out the existence of the solutions the main problem is to find the positive interval $[0,t_0](0 < t_0 \le 1)$ on which the (7) will take place.

From theorem 2.2 and quality (7) we can make the following consequence.

Consequence 2.1. If exists the solution of game Γ_F on the positive interval $t \in [0,t_0]\left(i \cdot e \cdot G\left(\overline{\Gamma}^L\right) \neq \phi\right)$, then this interval we must find from the solution of the inequality $\underline{v(t)} < \overline{v(t)}$..

Example 2.2. In the game Γ_F with payoff of matrix

$$F(t) = \begin{pmatrix} 1-t & t \\ t & 1-t \end{pmatrix}$$

near the zero $\underline{v(t)} = t, \quad \overline{v(t)} = 1-t$. From the inequality $t < 1-t$ we have $t < \frac{1}{2}, \quad i.e. \quad t \in \left[0, \frac{1}{2}\right]$. In the corresponding lexicographic game $\Gamma^L = \left(\Gamma^1, \Gamma^2\right)$ with payoff matrix

$$\mathrm{H} = \left(\mathrm{H}^1, \mathrm{H}^2\right) = \begin{pmatrix} (1,0) & (0,1) \\ (0,1) & (1,0) \end{pmatrix}$$

we have the solution $\left(X^*, Y^*\right) = \left(\left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}\right)\right)$, which is also the solution of game Γ_F on interval $t \in \left[0, \frac{1}{2}\right]$.

The function of payoff in the situation $(X,Y)\in \aleph_1\times\aleph_2$ with payoff matrix $\mathrm{H}=(\mathrm{H}^1,...,\mathrm{H}^m)$ in the lexicographic $p\times q$-matrix game $\Gamma^L=(\Gamma^1,...,\Gamma^m)$ in view of (5) is equal:

$$XF(t)Y^T=F(t)(X,Y)=H^1(X,Y)+\sum_{k=2}^{m}(H^k-H^{k-1})(X,Y)t^{k-1}, \qquad (8)$$

where

$$\mathrm{H}^k(X,Y)=\sum_{i=1}^{p}\sum_{j=1}^{q}a_{ij}^k x_i y_j,\quad (H^k-H^{k-1})(X,Y)=H^k(\mathrm{X},\mathrm{Y})-\mathrm{H}^{k-1}(\mathrm{X},\mathrm{Y})=$$
$$=\sum_{i=1}^{p}\sum_{j=1}^{q}\left(a_{ij}^k-a_{ij}^{k-1}\right)x_i y_j,\quad k=1,\cdots,m.$$

For (8) function the formula of Taylor has such form:

$$F(t)(X,Y)=\sum_{k=0}^{m-1}\frac{F^{(k)}(0)(X,Y)}{k!}t^k,$$

where

$$F^{(0)}(0)(X,Y)=H^1(X,Y),\frac{F^{(k)}(0)(X,Y)}{k!}=(H^{k+1}-H^k)(X,Y),$$
$$k=1,...,m-1.$$

Theorem 2.3. If (X^*,Y^*) is a saddle point in every game $\Gamma^1,\Gamma^2-\Gamma^1,...,\Gamma^m-\Gamma^{m-1}$, then

$$(X^*,Y^*)\in G(\overline{\Gamma}_F[0,1]) \text{ and } (X^*,Y^*)\in G(\overline{\Gamma}^L).$$

<u>Proof.</u> By the condition of theorem for every $X\in\aleph_1$ *and* $Y\in\aleph_2$ it has such form:

$$\mathrm{H}^1(X,Y^*)\le\mathrm{H}^1(X^*,Y^*)\le\mathrm{H}^1(X^*,Y),$$

$$(\mathrm{H}^k-\mathrm{H}^{k-1})(X,Y^*)\le(\mathrm{H}^k-\mathrm{H}^{k-1})(X^*,Y^*)\le(\mathrm{H}^k-\mathrm{H}^{k-1})(X^*,Y),$$
$$k=2,\cdots,m.$$

If we'll multiply these inequalities according to $1,t,\cdots,t^{m-1}$ and add each other, in vew of (8) we'll get that for every $t\in[0,1]$ the following inequalities will take place:

$$F(t)(X,Y^*)\le F(t)(X^*,Y^*)\le F(t)(X^*,Y),\qquad \forall X,\quad \forall Y.$$

This means, that $(X*,Y*)\in G(\overline{\Gamma}_F)$ for every $t\in[0,1]$. In view of (6) it carry out that $(X*,Y*)\in G(\overline{\Gamma}^L)$.

Hence for Γ^L and Γ_F matrix games the inclusions take place

$$G(\Gamma^1)\bigcap_{k=2}^{m} G(\Gamma^k-\Gamma^{k-1})\subseteq G(\overline{\Gamma}_F)\subseteq G(\overline{\Gamma}^L),$$

or

$$\bigcap_{k=0}^{m-1} G(\overline{\Gamma}_{F^{(k)}(0)})\subseteq G(\overline{\Gamma}_F)\subseteq G(\overline{\Gamma}^L).$$

Note 2.1. It is not necessary to fulfil $(X*,Y*)\in G(\overline{\Gamma}^L)$ in the conditions of Theorem 2.3. For example, in the lexicographic game Γ^L with payoff matrix

$$\mathrm{H}=(\mathrm{H}^1,\mathrm{H}^2)=\begin{pmatrix}(2,1) & (3,0)\\ (1,2) & (2,2)\end{pmatrix}$$

we have $G(\Gamma^L)=(1,1)$. In the corresponding game Γ_F the matrix of payoff is

$$F(t)=\begin{pmatrix}2 & 3\\ 1 & 2\end{pmatrix}+\begin{pmatrix}-1 & -3\\ 1 & 0\end{pmatrix}\cdot t.$$

Evidently, that

$$G(\overline{\Gamma}^1)=(1,1),\quad G(\overline{\Gamma}^2-\overline{\Gamma}^1)=(0,0),\quad G(\overline{\Gamma}^1)\bigcap G(\overline{\Gamma}^2-\overline{\Gamma}^1)=\phi$$

2). Now let us consider the case, when the matrix elements $f_{ij}(t)$ of payoff in the matrix game Γ_F are more general functions of one t current which have uninterrupted derived in every kind of row on the interval $[0,1]$.

Let us take one more modification of saddle point for Γ_F matrix game. Let's the $\{t_n\}$ sequence from $[0,t_0]\subset[0,1]$ interval of the positive numbers such that $\lim_{n\to\infty} t_n=0$.

Definition 2.3. The situation (X^*, Y^*) in game Γ_F with payoff of matrix $F(t)$ is called a saddle point (or a solution) on $\{t_n\}$, if for every $t \in \{t_n\}$ from the beginning of *n* will fulfil (4). At this time we'll write $(X^*, Y^*) \in G(\overline{\Gamma}_F\{t_n\})$.

It is easy to show, that $G(\overline{\Gamma}_F[0, t_0]) \subseteq G(\overline{\Gamma}_F\{t_n\})$.

Theorem 2.4. If it exists such sequence $\{t_n\} \subset [0, t_0]$ that $\lim_{n\to\infty} t_n = 0$ and every scalar game $\Gamma_{F(t_n)}$ $(n = 1,2,...)$ are equivalent strategically, then $G(\overline{\Gamma}_F\{t_n\}) \neq \phi$.

The conclusion is carring out from the definition of the strategic equivalence of the scalar games

Let us spread every $f_{ij}(t)$ of matrix $F(t)$ as the row of Maklaurin in the $t = 0$ point:

$$\tilde{f}_{ij}(t) = \sum_{k=0}^{\infty} \frac{f_{ij}^{(k)}(0)}{k!} t^k, \quad i = 1,..., p; \quad j = 1,..., q$$

and consider the matrix game Γ_F with payoff matrix $\tilde{F}(t)$:

$$\tilde{F}(t) = \{\tilde{f}_{ij}(t)\} = \left\{ \sum_{k=0}^{\infty} \frac{f_{ij}^{(k)}(0)}{k!} t^k \right\}.$$

Let us consider infinite dimensional lexicographic pxq-matrix game $\Gamma^L{}_{(\infty)} = (\Gamma^1, \Gamma^2, ...)$ with payoff matrix (Beltadze, 1991)

$$\mathrm{H} = (\mathrm{H}^1, \mathrm{H}^2, ...) = \{(a_{ij}^1, a_{ij}^2, ...)\}.$$

For its affine matrix game $\Gamma_{(t,\infty)}$ by analogy on the game (5) has matrix of payoff

$$\mathrm{H}_{(t,\infty)} = \mathrm{H}^1 + \sum_{k=2}^{\infty} (\mathrm{H}^k - \mathrm{H}^{k-1}) t^{k-1},$$

where

$$\mathrm{H}_{(t,\infty)} = \{a_{ij}(t)\}, \quad a_{ij}(t) = a_{ij}^1 + \sum_{k=2}^{\infty} (a_{ij}^k - a_{ij}^{k-1}) t^{k-1}, i = 1,..., p; j = 1,..., q,$$

i.e. the element $a_{ij}(t)$ of the matrix $\mathrm{H}_{(t,\infty)}$ has the form of the function $\tilde{f}_{ij}(t)$.

Hence, on the basis of matrix H of the payoff of $\Gamma_{(\infty)}$ game we have done the payoff matrix $\tilde{F}(t)$ with $\tilde{f}_{ij}(t)$ elements. Now, on the contrary let us suppose we have the matrix $\tilde{F}(t)=\{\tilde{f}_{ij}(t)\}$. Let us note

$$a_{ij}^1=\tilde{f}_{ij}(0),\, a_{ij}^2-a_{ij}^1=\frac{\tilde{f}_{ij}^1(0)}{1!},\ldots,a_{ij}^m-a_{ij}^{m-1}=\frac{\tilde{f}_{ij}^{(m-1)}(0)}{(m-1)!},\ldots$$

From these we'll denote $a_{ij}=(a_{ij}^1,a_{ij}^2,\ldots)$ and we'll get matrix H. Hence, as in the cases of the polynomial payoffs, here we can consider the following theorem.

Theorem 2.5. The matrixes of payoff and $\tilde{F}(t)$ are denoting each other one valued.

Note 2.2. If every row $\tilde{f}_{ij}(t)$ of matrix $\tilde{F}(t)$ is converging on any positive interval $[0,t_0]$towards $f_{ij}(t)$ function, i.e. if $\tilde{F}(t)=F(t)$, then between $F(t)$ and H matrix exists one valued relation.

Example 2.3. Γ_{F} game payoff's matrix has the following form:

$$F(t)=\begin{pmatrix} e^{-t} & -t \\ t & 1+t \end{pmatrix}.$$

Let us find the corresponding lexicographic Γ^L game payoff's matrix $\mathrm{H}=\{a_{ij}\}$.

Let's spread $f_{11}(t)=e^{-t}$ function as the row of Maklaurin in the $t=0$ point:

$$e^{-t}=1-\frac{t}{1!}+\frac{t^2}{2!}-\frac{t^3}{3!}+\ldots+(-1)^n\frac{t^n}{n!}+\ldots\ .$$

Let us find the corresponding Γ^L game payoff's H matrix $a_{11}=(a_{11}^1,a_{11}^2,\ldots)$ element. As

$$a_{11}^1=1,a_{11}^2-a_{11}^1=-\frac{1}{1!},a_{11}^3-a_{11}^2=\frac{1}{2!},a_{11}^4-a_{11}^3=-\frac{1}{3!},\ldots,a_{11}^n-a_{11}^{n-1}=(-1)^{n-1}\frac{1}{(n-1)!},\ldots\ ,$$

therefore we find

$$a_{11}=\left(1,0,\frac{1}{2!},\frac{2}{3!},\frac{9}{4!},\ldots\right).$$

H matrix's a_{12},a_{21} and a_{22} elements' we'll find easily. In fact, as

$$f_{12}(t)=-t=0+(-1)t+0.t^2+\ldots\ ,$$

therefore $a_{12}^1 = 0, a_{12}^2 - 0 = -1, a_{12}^3 - a_{12}^2 = 0, \ldots,$ and $a_{12} = (0,-1,-1,-1,-1,\ldots)$.

Analogically $a_{21} = (0,1,1,1,1,\ldots)$, $a_{22} = (1,2,2,2,2,\ldots)$. Hence, matrix Γ^L game payoff's matrix is

$$\mathrm{H} = \left(\mathrm{H}^1, \mathrm{H}^2, \ldots\right) = \begin{pmatrix} \left(1,0,\frac{1}{2!},\frac{2}{3!},\frac{9}{4!},\ldots\right) & (0,-1,-1,-1,-1,\ldots) \\ (0,1,1,1,1,\ldots) & (1,2,2,2,2,\ldots) \end{pmatrix}.$$

Hence, in view of the 2.5 th theorem from matrix game Γ_{F} with matrix $F(t)$ of payoff, we can make the lexicographic matrix game $\Gamma^L{}_{(\infty)}$ with payoff matrix:

$$F_{(\infty)}(0) = \left(\{f_{ij}(0)\}, \left\{ f_{ij}(0) + \frac{f_{ij}^1(0)}{1!} \right\}, \left\{ f_{ij}(0) + \frac{f_{ij}^1(0)}{1!} + \frac{f_{ij}^{11}(0)}{2!} \right\}, \ldots \right).$$

Let us note

$$F_{(\infty)}(0) = \left(F^1(0), F^2(0), \ldots, F^m(0), \ldots\right),$$

where

$$F^k(0) = \left\{ f_{ij}(0) + \frac{f_{ij}^1(0)}{1!} + \ldots + \frac{f_{ij}^{(k-1)}(0)}{(k-1)!} \right\}, k = 1,2,\ldots$$

In view of the theorem 1 (Beltadze, 1991) for the lexicographic matrix game $\Gamma^L{}_{(\infty)}$ with matrix of payoff $F_{(\infty)}(0)$ we have

$$G\left(\overline{\Gamma}^L{}_{(\infty)}\right) = \bigcup_{s \in (0,1]} \bigcap_{\tau \in [0,s]} G\left(\lim_{m \to \infty} \overline{\Gamma}_{F_m(\tau)}\right),$$

where $\Gamma_{\mathrm{F}_{(\mathrm{m})}(\tau)}$ is pxq-matrix game with matrix of payoff

$$F_{(m)}(\tau) = F^1(0) + \left(F^2(0) - F^1(0)\right)\tau + \cdots + \left(F^m(0) - F^{m-1}(0)\right)\tau^{m-1}.$$

For the $\Gamma^L{}_{(\infty)}$ game from theorem 2 (Beltadze, 1991) in this case, we may form in this way: suppose on the $\aleph_1 \times \aleph_2$ set of payoff matrix $F_{(\infty)}(\tau)$ of Maklaurin rows are converging on any positive interval $\tau \in [0, t_0]$. In order, that $\left(X^*, Y^*\right) \in G\left(\Gamma_F[0, t_0]\right)$ is necessary and sufficient to fulfil the following inequalities

$$\lim_{m\to\infty} F_{(m)}(\tau)(X,Y^*)\le \lim_{m\to\infty} F_{(m)}(\tau)(X^*,Y^*)\le \lim_{m\to\infty} F_{(m)}(\tau)(X^*,Y)$$

for any $X\in\aleph_1$ and $Y\in\aleph_2$ on the interval $\tau\in[0,t_0]$.

The payoff of the first player in the matrix game Γ_F in the situation (X,Y) is equal

$$XF(t)Y^T \equiv F(t)(X,Y)=\sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=0}^{\infty}\frac{f_{ij}^{(k)}(0)}{k!}t^k x_i y_j=\sum_{k=0}^{\infty}\left(\sum_{i=1}^{p}\sum_{j=1}^{q}\frac{f_{ij}^{(k)}(0)}{k!}x_i y_j\right)t^k,$$

which is consider, that is equally convering on $[0,t_0]$ for every $(X,Y)\in\aleph_1\times\aleph_2$. This equality is the same, as

$$F(t)(X,Y)=\sum_{k=0}^{\infty} XF_kY^T\cdot t^k=\sum_{k=0}^{\infty} F_k(X,Y)t^k=\sum_{k=0}^{\infty}\frac{F^{(k)}(0)(X,Y)}{k!}t^k,$$

where

$$F_k=\left\{\frac{f_{ij}^{(k)}(0)}{k!}\right\},\quad \frac{F^{(k)}(0)(X,Y)}{k!}=F_k(X,Y),\quad k=0,1,2,\ldots$$

Consequence 2.2. The following inclusion takes place

$$\bigcap_{k=0}^{\infty} G(\overline{\Gamma}_{F_k})\subseteq G(\overline{\Gamma}_F[0,t_0]).$$

3. CONCLUSIONS

We are having into a practice the necessity of projecting an economical, political and technical systems, when for obtaining the global aim must be carry out different kinds of private aims' successive attainment. At the same time, the degree of obtaining each private aim is defined with corresponding criterium. In that case criteria with the help of them we choose the variants of the system are ranked strictly. Strict ranking of the criteria causes the lexicographic preference'c relation to the set of the variants of the system and vice versa. Lexicographic $\Gamma^L=(\Gamma^1,\ldots,\Gamma^m)$ game's payoff $H=(H^1,\ldots,H^m)$ vector-function criteria $H^1,\ldots,H^m$ are ranked strictly. The antagonistic Γ_F game given in the article with functional elements, shows that in the process of the sides' contrapositive and in the game against the nature, for getting the certain aim, when the preference of the player changes in time and it is necessary to choose the time's starting moments, making decision is lead to using the strictly ranked criteria. If Γ_F

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payoff's $F(t)$ matrix elements are polynomials of the finite degree, then the number of such criteria is finite, but even it one element isn't polinomial, then number of private criteria is nonfinite.

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