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THE MODEL OF THE LINEAR CITY UNDER TRIANGULAR DISTRIBUTION OF CONSUMERS

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Abstract
This paper presents a model of oligopolistic competition in presence of horizontal differentiation of goods, under a triangular distribution of consumers. The triangular distribution represents a case of concentration of consumers around the central location. The main result is that a good deal of differentiation among goods can be achieved in equilibrium also under such assumption concerning the consumers' distribution. This means that the incentive to differentiate prevails on the incentive to hold the central location, even if consumers are concentrated in the central location. The analysis based on an empirical case-study is presented, concerning the choice of beverage retails in a town. The empirical evidence is consistent with the theoretical model.
Keywords: orizontal product differentiation, Hotelling, maximum differentiation, minimum differentiation.

1. INTRODUCTION

In this paper I analyze a duopoly under endogenous horizontal differentiation. An intuitive approach to this problem is represented by *location models* which starts from Hotelling's seminal contribution in 1929. In his model, Hotelling (1929) argues that the utility function may assume different levels among consumers according to their location over the [0,*l*] delimited linear space. Hotelling's result, achieved under uniform distribution of consumers, is as follows: in a game where firms choose product varieties, expecting to receive the equilibrium profits as pay-off, they (firms) decide to produce extremely similar products. This phenomenon is called "Principle of Minimum Differentiation".

Since Hotelling (1929) a vast literature concerning product differentiation - in terms of spatial competition - has been developed. In short, this literature shows that the equilibrium as defined by Hotelling is very weak because firms, for given locations, find more profitable to adopt *undercutting strategies*.

A significant contribution to Hotelling (1929)'s model was given by D'Aspremont, Gabszewicz and Thisse (1979). They show that, within such a market, the problem of non-existence of a non-cooperative equilibrium in the price stage arises from the fact that consumers located – on the basis of their preferred variety – close to market's edges are captured by their closest firm for a large range of prices but *not* for every price. Indeed, at some (low) level of price, these consumers are 'lured' by the distant firm. This circumstance creates

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incentives to *expel* from the market the opponent firm in order to be monopolist and, in turns, it makes unfeasible the existence of an equilibrium in prices.

D'Aspremont, Gabszewicz e Thisse (1979), by means of the introduction of a quadratic cost function, solve this problem achieving the "Maximum Differentiation Principle": firms fix their product's specification at the opposite sides of the market.

Economides (1986) enounces more general conditions concerning the effects of the transportation cost function on the market equilibrium.

The purpose of this paper is to investigate the robustness of the "Maximum Differentiation Principle", using a different consumers' distribution. Indeed, in Hotelling (1929)'s model consumers are assumed to be uniformly distributed, but in the real world often consumers tastes are gathered around a central value of a specific product characteristic. In addition, thinking to location purely in a geographical sense, we expect to observe people concentrated toward the central location[1].

The novelty introduced by the present paper is the introduction of a triangular distribution of consumers to stress this occurrence. The analysis shows that, even if consumers are gathered around the central location, minimum differentiation could be not profitable[2].

The final part of the paper presents empirical evidence concerning location-choice and price-choice of beverage kiosk in Catania (Italy). The analysis focuses on a refreshing drink, made using water, lemon and mandarin which is a traditional commodity of Catania's folk culture. Since the productive process is very simple, two drinks may differ for their sale location only, just as assumed by models here considered. This analysis shows that the central location might coexist with high price, by contrast with standard model result.

The paper is organized as follows. In section 2 preliminary results are summarized pointing out the relevance of transportation cost function to the equilibrium existence. In section 3 I introduce a model under triangular distribution of consumers presented as a particular parameter restriction of a set of trapezoidal distribution. Section 4 briefly shows data concerning empirical evidence about beverage retails in Catania. Some comments and concluding remarks are provided in section 5.

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1 For a general exposure of some of these models see: Carraro and Graziano (1993), Garella and Lambertini (2001) and in an optics related to the problems of urban geography Cori *et al.* (1998)

2 However, the validity of the Principle of Minimum Differentiation could be not completely excluded. Jehiel(1992), for example, shows that if firms play an infinite (or unknown) number of repetition of the two-stages game, they can collude in the price-stage. In this case, because of a *soft strategic-effect*, each firms chooses central location. This point will be discussed at the end of the analysis.

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2. SOME POINTS OF THE RELEVANT LITERATURE

In this section I describe the analytical framework regarding the basic model of location under uniform distribution.

First, I analyze the linear transportation cost function case. Then, I recall some results achieved in the quadratic transportation cost case. Some general conditions concerning equilibrium existence are also enounced at the end of this section.

To begin with, let me consider a linear city in the [0,1] interval delimited space. Within this city it is assumed the existence of a continuum of individuals differing in one dimension only: their location according to the type of commodity they like most. These individuals are uniformly distributed and each consumer takes at most one unit of the product which are produced at zero marginal costs.

Each consumer prefers to buy their good exactly where she/he is localised, so that, if a consumer has to go to a different location in order to purchase a good, she/he pays an additional *transportation cost*. Hence, a consumer (localized in *m* or) of type *m,* $0 \leq m \leq 1$, has an utility function expressed in monetary terms as

[2.1] $$U_m = s - p - f(d)$$

where *s* index the "gross" benefit deriving by consuming the good (i.e. if provided exactly in the consumer's location and regardless of the price paid for it) *p* is the price, *d* is the distance between *m* and the sell point x_i, that is, $d = |x_i - m|$, and *f*(*d*) is an increasing function of this distance. Moreover, $p + f(d)$ is the so-called *delivered price.*

This utility function may assume different forms; however it shows a peak when *p* and *d* are equal to zero and this peak value is *s*. This means that each consumer gains the same satisfaction if we ignore prices and transportation cost.

Economides (1986) proposed to express this transportation cost in the following form

[2.2] $$f(d) = t \cdot |x_i - m|^{\delta} \; ; \quad t > 0 \, , \quad 1 \leq \delta \leq 2$$

where the parameter *t* can be interpreted as a sensibility index of consumers' preferred specification, or, in a spatial way, it may be interpreted as a consumer's "idleness" index.

Note that if *t* is the same among consumers, this means that each consumer presents an identical sense of distance.

The parameter δ is used to introduce different form of transportation cost function. For example, a linear transportation cost can be obtained if $\delta = 1$, and also a quadratic function might be achieved by fixing $\delta = 2$.

Note that from [2.1] a condition arises, depending both on price and distance, that must be verified if *m*-th individual decide to purchase the commodity

[2.3] $U_m \geq 0 \Leftrightarrow s \geq p + f(d)$.

Thus, the *m*-th consumer gains an utility which is equal to total purchase cost. This condition has a crucial role within this market[3].

Before developing formal analysis, I provide an intuitive approach to these issues. Intuition behind the formal analysis may be explained as follows. We have to answer to these questions: if two firms cover this market producing homogeneous goods, central location is profitable for both firms? Or, by contrast, is it more profitable to maintain a certain distance from the rival one? And, finally, what we can say about prices fixed within this market?

Obviously, a central location not only provides an higher number of consumers (that is the so-called *demand-effect*), but also implies a strong price-competition (so-called *strategic-effect*). Thus, central location has two opposite effects. This simple argument shows the most important characteristic of this market: firm's behaviours are interdependent both in price-stage and location-stage.

Hotelling (1929) was the first one to use a spatial approach to this issue. Here I refer to Hay e Morris (1979)'s version of Hotelling's "Main Street" model.

According to considerations about interdependence mentioned above, the competition between firms is described by mean of a two-stage game. While in the second stage each firm use price as strategic variable, in the first stage firms choose their products' specifications (location) expecting to receive the payoff corresponding to the Nash equilibrium in prices strategies (i.e. given the second stage result).

In order to find the Nash equilibrium in the second stage of the game we need to the express the demand function for each firm. Clearly, this demand depends on their location. Let *a* and $1-b$ $(a, b \geq 0)$ be the firms' distance from 0, thus firm' location in [0,1] space will be $x_1 = a,\ x_2 = 1-b,\ x_1 \leq x_2$.

It is worth noting that if $a + b = 0$ we have the maximum degree of differentiation. If, instead, $a + b = 1$ firms are localised in the same point. Thus, on the first case we have a *soft strategic effect* and, by contrast, in the second one, firms face the maximum degree of price competition. In particular if firms choose the same location, their profit collapse to zero due to the *Bertrand competition* that drives the price differential over marginal cost to zero.

[3] In this one-dimension world, the demand function is very peculiar, since each consumer buys either zero or one unit of good; a neoclassical concept like "marginal utility", and its relationship with price, is pointless.

Given x_1, x_2 we are able to individuate the consumer who is indifferent between buying from firm 1 at price p_1 or from firm 2 at price p_2, because he gain the same utility. In fact, if both prices respect the [2.3] condition, in this market consumers have to choose between two possibility arising from two price and two transport cost, but it will be a consumer who is indifferent between the two possibility. Let *m** be the indifferent consumer as defined by following [2.4]

[2.4] $$p_1 + t \cdot |x_1 - m^*| = p_2 + t \cdot |x_2 - m^*|$$

Hence, all consumers on the left of *m** prefer firm 1 and, all consumers on the right of *m** prefer firm 2, moreover we obtain the demand of firm 1 and firm 2 by solving with respect to *m** equation [2.4] and substituting $x_1=a$ $x_2 = 1-b$.

The demand function for the two firms are respectively:

[2.5a] $$D_1(p_1, p_2) = a + \frac{1-a-b}{2} - \frac{p_1 - p_2}{2t}$$

[2.5b] $$D_2(p_2, p_1) = b + \frac{1-a-b}{2} - \frac{p_2 - p_1}{2t}.$$

It is also worth noting that each demand is constituted not only by a positive term which represents exactly its location (plus a term equal to the half of consumers who are contained between the two firms) but also by a negative term which shows the effect of the price differential. If considered in absolute value, this last term is increasing on the "new" price-variable defined as $p_j^{'} = p_j - p_i$ with $j \neq i$.

By observing the two [2.5] the parameter *t* can be interpreted as unity of measurement of price differential: the higher is 2*t* the lower will be $p_j^{'}/(2t)$.

Since [2.4] shows that each firm can enlarge its market share by fixing a price lower than the rival firm's price, each firm can have some incentive to lower her price to subtract market share to the rival one.

A possible strategy for every enterprise is then that to move the price downward to achieve such level that allows to serve alone the whole market (this level of price is well known as *price of exclusion*).

If a firm decide to cohabite with the other firm, it will be fixed a price higher than the exclusion price. Between the two possible (exclusion or not exclusion) strategies it will exist a price over which the price cannot go down, otherwise the market collapse in a monopoly.

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This limit-value conceptually corresponds to the case in which the indifferent consumer exactly coincides with the rival' location. To easily realize this, it is enough to consider that if the enterprise 1 fixes the price according to the rule:

[2.6] $$p_1 = p_2 - t(1-a-b)$$

the indifferent consumer would be exactly the consumer located in x_2. In effects, if the consumer with this location decided to purchase near the enterprise 1 (farer from him) such a price ($p_1 < p_2$) that integrally compensates the cost of would stop sustained transport. So, if the enterprise 1 sets a price that more than compensates the cost of transport, that is,

[2.7] $$p_1 < p_2 - t(1-a-b)$$

then any consumer would purchase from the enterprise 2.

In this way, through an opportune choice of the price, the enterprise 1 would send away from the market the enterprise 2 leading the market in a situation of monopoly.

The consequence is that to continue the oligopoly analysis the following condition must hold

[2.8] $$-t(1-a-b) \le p_2 - p_1 \le +t(1-a-b)$$

Therefore, equilibrium prices (p_1^*, p_2^*) of oligopoly game may exist only in the dominion defined by [2.8], in which both the enterprises have a positive market share.

Finally, should be noted that if prices satisfy [2.8], the indifferent consumer will be positioned between the two enterprises.

Each firm is interested in maximizing profit function represented by a function ($\Pi_i(a,b) = p_i\,(a,b) D_i(a,b,p_i\,(a,b).p_j(a,b))$) that depends on the decisions on location and price of both the enterprises.

By differentiating the firms' profit functions and solving the first order condition with respect to prices, we find reaction functions:

[2.9] $$\begin{cases} \dfrac{\partial \Pi_1}{\partial p_1} = 0 \Rightarrow p_1 = \dfrac{t}{2}(1+a-b) + \dfrac{1}{2} p_2 \\ \dfrac{\partial \Pi_2}{\partial p_2} = 0 \Rightarrow p_2 = \dfrac{t}{2}(1-a+b) + \dfrac{1}{2} p_1 \end{cases}$$

The Nash equilibrium in prices is therefore:

[2.10a] $$p_1^*(a,b) = t\left(1 + \frac{a-b}{3}\right)$$

[2.10b] $$p_2^*(a,b) = t\left(1 - \frac{a-b}{3}\right)$$

equilibrium prices have the following propery: they are increasing on *t*, coherently with the circumstance that higher levels of t "damp" the price effect on the demand function: if the distance is relatively less important than the price, prices are pushed upward and vice versa; they decrease, instead, when the enterprises are close, because of this involves a sourer price competition

By substituting equilibrium price into profit function we obtain the following equilibrium profits

[2.11a] $$\Pi_1^* = \frac{t}{2}\left(1 + \frac{a-b}{3}\right)^2$$

[2.11b] $$\Pi_1^* = \frac{t}{2}\left(1 - \frac{a-b}{3}\right)^2$$

in order the couple (p_1^*, p_2^*) to constitute a Nash equilibrium, it is not sufficient that it assures the profit maximization within the interval defined by [2.9], but it has to generate a level of profit higher than the profit that firm would get in monopoly, excluding the rival. Thus, the following conditions must be verified

[2.12a] $$\Pi_1(p_1^*, p_2^*) = \frac{t}{2}(1 + \frac{a-b}{3})^2 \geq (p_2^* - t(1-a-b) - \varepsilon) = \Pi_1^m$$

[2.12b] $$\Pi_2(p_1^*, p_2^*) = \frac{t}{2}(1 - \frac{a-b}{3})^2 \geq (p_1^* - t(1-a-b) - \varepsilon) = \Pi_2^m$$

Clearly, until both firms are present in the market, it does not exist a reciprocally best choice which is different to the couple (p_1^*, p_2^*); the only alternative credible strategy would consist in expelling from the market the rival firm, and it is exactly the convenience of this strategy that we have to study.

Undoubtedly, each firm is interested in serving the whole market, by practicing the highest price. Thus, we evaluate what happens for $\varepsilon \to 0$, studying the system of the following conditions

[2.13a] $$\left(1 + \frac{a-b}{3}\right)^2 \geq \frac{4}{3}(a + 2b)$$

[2.13b] $$\left(1+\frac{b-a}{3}\right)^2 \geq \frac{4}{3}(b+2a)$$

Conditions [2.13a,b] imply that the couple (p_1^*, p_2^*) represents a situation of equilibrium in which it is convenient the cohabitation with the other enterprise, only for specific configurations of the possible relative positions of the two firms (that is, for specific parameters combinations)

Moreover, conditions [2.13a,b] on the location are necessary and sufficient for the existence of the equilibrium in oligopoly, since if they are satisfied, condition [2.8] will be also satisfied.

The conclusion is that prices (p_1^*, p_2^*) that bring the market into an equilibrium with positive profits for both enterprises, do not induce firms to look for eliminate the competitors from the market, only for a subset of the first-stage selected positions.

In order to answer to the question on the location in oligopoly, it will be enough to solve the profit maximisation first order conditions with respect to *a* and *b* respectively. In fact, *a* and *b* are the strategic variables of the decisional node here considered. In this way we obtain

[2.14a] $$\frac{\partial \Pi_1^*}{\partial a} = \frac{t}{3}\left(1+\frac{a-b}{3}\right) > 0$$

[2.14b] $$\frac{\partial \Pi_2^*}{\partial b} = \frac{t}{3}\left(1-\frac{a-b}{3}\right) > 0$$

At a first glance, it could seem that in this market a natural force exists, leading firms to converge toward the centre, up to place both exactly in the central position. But, it has already been noticed that the profit function is not continuous in *a* and *b*, since it presents a discontinuity for *a*+*b*=1: in such a case prices equal the marginal cost..

This circumstance implicates the existence of a tension among the two enterprises that are pushed to estrange from the centre to have positive profits. Yet, they do not move at all toward an equilibrium: in fact, or they get further in such way that conditions [2.13] is satisfied, and then it becomes convenient for both to move once more toward the centre, following the rule drawn by [2.14]; or, by contrast, they get further without [2.13] conditions are respected and in this case we have already shown that an equilibrium doesn't exist.

Therefore, we can conclude that, in the first stage of the game concerning location, no equilibrium exists in pure strategies, so that a sub-game perfect equilibrium does not exist in the whole game. To avoid this drawback, it is possible to modify some hypotheses introduced concerning. the parameters, without shaking the general framework.

A possible way to operate consists in removing the hypothesis of linearity of the transport function, simply setting $\delta = 2$ into [2.2]. The transportation cost function so modified assumes the form

[2.15] $f(d) = t(x_i - m)^2$

The point made by D'Aspremont, Gabszewicz and Thisse (1979) can be reassumed in the following terms: if the equilibrium problem, comes from the fact that when firms are next to the centre, small reductions of price they allow to serve the whole market, then the solution is to eliminate this discontinuity in profit function.

This change imposes to reconsider both the stadiums of the game.

New indifference condition becomes[4]:

[2.16] $p_1 + t(m-a)^2 = p_2 + t(1-b-m)^2$

Solving with respect to *m* this new demand functions we obtain the modified equilibrium conditions. Focus one attention to the first stage where firms choose their position, the following [2.17] shows that if $0 \le a \le 1-b \le 1$ is hold profit function derivative are negative on *a* and *b* respectively for firm 1 and firm 2. Or rather, in the second stage of the game both oligopolists are interested to get further and to position themselves into the two opposite edges of the market, because of the profit function decreases in *a* and *b* respectively.

[2.17a] $\frac{\partial \Pi_1}{\partial a} = p_1^* \left(\frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_2} \frac{\partial p_2}{\partial a} \right) < 0$

[2.17b] $\frac{\partial \Pi_2}{\partial b} = p_2^* \left(\frac{\partial D_2}{\partial b} + \frac{\partial D_2}{\partial p_1} \frac{\partial p_1}{\partial b} \right) < 0$

The existence of equilibrium strategies for both stage of the game assures, in last, also the existence of a subgame perfect equilibrium in the whole game. It emerges, therefore, in a rather predictable way, the role of the transportation function as element which is able to influence the existence itself of the equilibrium.

Economides (1986) shows, in fact, that from different values assumed by δ into [1,2] the existence or inexistence of equilibrium strategies arises, according to the following rule:

[4] This follows from the fact that derivative of m^2, i.e. $2m$, is never equal for two different values of *m* of the same sign, so that generalised price curves can cross, but never coincide, given the firms are not located at the same point.

- If $\delta \in$[1; 1,26] perfect equilibrium doesn't exist in the pure strategies of the game;
- if $\delta \in$[5/3; 2] a subgame perfect equilibrium exists in which $p_1^*(a,b,\delta), p_2^*(a,b,\delta)$ are

  a Nash equilibrium in the price-stage and in the location-stage the principle of the maximum differentiation is satisfied.
- if $\delta \in$[1,26; 5/3], finally, a subgame perfect equilibrium exists. Equilibrium locations are defined by $x_1^*(\delta) = \frac{5-3\delta}{4}; \quad x_2^*(\delta) = 1 - x_1^*(\delta) = \frac{3\delta - 1}{4}$.

Thus, in the third case, $x_1^* > 0 \quad x_2^* < 1$, it is an open question whether we observe a weak demonstration of the maximum or minimum differentiation principle.

Until this point the hypothesis of uniform distribution of consumer along the line segment representing the market has been maintained. Nevertheless, this kind of distribution it is not a very likely one.

Therefore, next section proposes to modify the model considering a different (and more realistic) consumers' distribution. A quadratic transportation cost hypothesis is maintained.

3. THE BASIC-MODEL UNDER TRIANGULAR CONSUMERS' DISTRIBUTION

In the real world often we observe distributions which have a peak around their central value, also when they refer – and that it is the intuition of models here considered – to preferences around one determined product characteristic.

The distribution that I introduce is triangular and does not change total market dimension; even recalling the form of the *normal distribution*, it does not exactly reproduce its form because it is more angular; nevertheless, it is evident that this difference regarding their form, does not change the intuition behind them: triangular distribution, with a great degree of analytical simplicity, represents a population "thickened" around the central location[5].

A distribution with this form has as density function as the following [3.1]

[3.1] $$f(x) = \begin{cases} 4m & per \quad 0 \le m \le \frac{1}{2} \\ 4 - 4m & per \quad \frac{1}{2} < m \le 1 \end{cases}$$

therefore, the corresponding distribution function is

[5] On property of triangular distributions see Johnson et al. (1995) and among recent applications of this distribution see Benassi, Cellini and Chirco (1999); and in reference to location problems see Scrimitore (2005).

[3.2] $$F(m)=\begin{cases} 2m^2 & per \quad 0\le m\le \frac{1}{2} \\ -2m^2+4m-1 & per \quad \frac{1}{2}<m\le 1 \end{cases}$$

In the following analysis, I try to answer this central question: does this different consumers' distribution imply some effect on equilibrium? Or rather: should we expect that the two firms will certainly place themselves in the central position, where a great density is observed, or, however, exists a possibility that a certain degree of differentiation inside this market is maintained?

Preliminarily, as noticed by Scrimitore (2003), it is instructive to observe that [3.1] represents a particular case of a whole set of density functions that describe distributions type trapezoidal with a elevated degree of concentration around the central position up to be reduced to a triangle[6].

Distributions of this type are defined by the [3.3] following

[3.3] $$f(m,\varphi)=\begin{cases} \frac{4}{1-\varphi^2}m, m<\frac{1-\varphi}{2} \\ \frac{2}{1+\varphi}m, \frac{1-\varphi}{2}<m<\frac{1+\varphi}{2} \\ \frac{4}{1-\varphi^2}m, m<\frac{1+\varphi}{2} \end{cases}$$

Hence, consumers, indexed with *m*, are distributed over the interval [0;1] according to a density function $f(m,\varphi)$ where the parameter φ can be interpreted as a concentration index of the consumers' tastes, or simply as a consumers' concentration index. If φ is equal to 1, then [3.3] describes a uniform distribution. If, by contrast, φ is equal to zero the function collapses into a triangle. And, also, as φ decreases, tending to zero, the distribution function concentrates toward the middle becoming trapezoidal. So, under the distribution described by [3.3] the number of the consumers contained into an interval vary depending on this interval's extremes.

To define the "new" demand function it is necessary to calculate the number of consumers contained in such interval. Therefore, let *a* and *b* denote respectively the distance of the firm 1 and 2 from the origin. Consequently

[3.4] $$m^*=\frac{1}{2}[\frac{p_2-p_1}{b-a}+b-a]+a$$

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[6] Note that this density is a trapezoid, with longest base equal to 1, shortest base equal to φ w and altitudo equal to $\frac{2}{1+\varphi}$ and it is also easy to check that if $\varphi=0$ is hold [3.3] became equal to the [3.1].

is the location of the indifferent consumer (and supposing as above that it him is localised between the two firms) the demand functions become $D_1 = F(m,\varphi)$ and $D_2 = 1 - F(m,\varphi)$, which can be written in the following way

[3.5a]

$$D_1 = \frac{\frac{p_2 - p_1}{b-a} + b + a + \frac{1}{2}(\varphi - 1)}{1+\varphi}$$

$$D_2 = 1 - \frac{\frac{p_2 - p_1}{b-a} + b + a + \frac{1}{2}(\varphi - 1)}{1+\varphi}$$

It is worth noting that it is possible to see a *strategic effect* and a *demand effect* in this market, by observing that D_1 not only has a positive term which represents location of firm 1 ("+*a*"), but also has a positive term which represents the price differential ($p_2 - p_1$) discounted by a factor that expresses the intervening distance between them (*b*-*a*). Thus, it is easy to check that if *a* tends to *b*, then $\frac{p_2 - p_1}{b-a}$ tends to infinity attributing (or subtracting if $p_2 < p_1$) infinite market shares: in this way the strategic role of prices may "explode"[7].

A last comment on $(\varphi - 1)$: it can be interpreted as an index of the divergence from the uniform distribution that directly acts to modify the demand function.

Once obtained the demand functions we obtain the following profit functions:

[3.6a] $$\pi_1 = p_1 \frac{\frac{p_2 - p_1}{b-a} + b + a + \frac{1}{2}(\varphi - 1)}{1+\varphi}$$

[3.6b] $$\pi_2 = p_2[1 - \frac{\frac{p_2 - p_1}{b-a} + b + a + \frac{1}{2}(\varphi - 1)}{1+\varphi}]$$

from them the following reaction functions can be individualized

[3.7a] $$p_1 = \frac{1}{2}[p_2 + b^2 - a^2 - \frac{1}{2}(a + b + \varphi(b-a))]$$

[3.7b] $$p_2 = \frac{1}{2}[p_1 - b^2 + a^2 - \frac{1}{2}(3(a+b) + \varphi(b-a))]$$

[7] If $a \to b \Rightarrow \frac{p_2 - p_1}{b-a} \to \infty$ and sign is given by $(p_2 - p_1)$.

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the price-stage has therefore an equilibrium (p_1^*, p_2^*) with

[3.8a] $$p_1^* = \frac{1}{3}(b^2 - a^2) + \frac{1}{6}(b-a) + \frac{1}{2}\varphi(b-a)$$

[3.8b] $$p_2^* = \frac{1}{3}(a^2 - b^2) + \frac{5}{6}(b-a) + \frac{1}{2}\varphi(b-a)$$

The reaction functions and the consequent equilibrium prices provide us with an interesting indication: as the consumers' distribution estranges from the uniform distribution it is on record that prices have a tendency to the rebate. To show this, note that the term $\frac{1}{2}\varphi(b-a)$ in which the distance between the two enterprises is damped from φ up to the total zero resetting of the addendum in the case of triangular distribution.

In the location-stage we need to find the positions that reciprocally constitute the best answer to the rival firm's profit maximizing behaviour (given prices (p_1^*, p_2^*)).

Since profit functions, given (p_1^*, p_2^*), are the following [3.9]

[3.9a] $$\pi_1^* = \frac{1}{6}[\frac{1}{3}(b^2 - a^2) + \frac{1}{6}(b-a) + \frac{1}{2}\varphi(b-a)]\frac{2(b+a)+1+3\varphi}{1+\varphi}$$

[3.9b] $$\pi_2^* = \frac{1}{6}[\frac{5}{6}(b-a) + \frac{1}{2}\varphi(b-a) - \frac{1}{3}(a^2 - b^2)]\frac{5+3\varphi-2(a+b)}{1+\varphi}$$

reaction functions with respect to location are represented by

[3.10a] $$a^* = \frac{1}{3}b - \frac{1}{2}\varphi - \frac{1}{6}$$

[3.10b] $$b^* = \frac{1}{3}a + \frac{1}{2}\varphi + \frac{5}{6}$$

Uniform distribution ($\varphi = 1$) estranges firms the one from the other, while if the degree of concentration increases, a tendency to draw near will be recorded. The optimal location for firms turns out to be

[3.11a] $$a^* = \frac{1}{8} - \frac{3}{8}\varphi$$

[3.11b] $$b^* = \frac{7}{8} + \frac{3}{8}\varphi$$

Therefore, the choice of central position does not emerge (i.e. $a^* \neq b^* \neq 1/2$) thus we have forces that lead the duopolist to maintain product differentiation.

For completeness, the equilibrium prices can be computed as follow

[3.12] $$p_1^* = p_2^* = \frac{3}{8} + \frac{3}{4}\varphi + \frac{3}{8}\varphi^2$$

The indifferent consumer is, finally, located in m = ½.

So, even in the limit-case $\varphi = 1$ that describes the triangular distribution, firms have convenience to earn the central position; indeed, the equilibrium involves location on the tails of the market.

This result is not intuitive: high central density would make profitable to localize around the middle in a way to exploit in a maximum measures the demand effect What is recorded, instead, is a light movement toward the centre; this firm's behaviour implies that an high degree of differentiation inside this market is preserved.Can we state that the central location will never be chosen? In reality it is necessary to underline the hypothesis that firm are not able to collude. Nevertheless, it is evident, that this is a very strong hypothesis not always realized.

Jehiel (1992) defines, for instance, a scenery within, if the game is repeated an endless number of times, firms are able to collude on price and to get elevated profits through elevated prices; the central position is so preferred not owing to fear the strategic effect.

4. AN EMPIRICAL ANALYSIS ON THE KIOSKS IN CATANIA

In this Section I propose an analysis on the most important drink kiosks in Catania[8].

While theoretical models distinguish the location stage from the price stage, in the real world, only the final outcome can be observed. Thus, I will try to "photograph" this result trying also to reconstruct the underlying dynamics.The case concerns to location choice of the drink kiosks in the city of Catania. I have noticed (in May 2005) location and practiced prices of more diffused products.

The location is situated (in very precise way) on two axis: Via Umberto-axis and Via Etnea-axis. On the first axis the kiosks of Piazza Iolanda, Via Umberto and Piazza Trento are established. On the Via Etnea-axis we found those of Piazza Santo Spirito, Piazza Carlo Alberto, Piazza Borsa and Via Santa Maddalena. The

[8] Kiosks here considered are located in some points that clearly hold different activities tied to the modern concept of "city": the kiosk of Piazza Spirito Santo has as basin of use the so-called "City" of Catania glanced the presence of different financial institutions, the two kiosks of Piazza Borsa and S.Agata La Vetere serve the tied up zone to the Chamber of Commerce and of the Faculty of Low. Kiosks located in "Piazza Carlo Alberto" have how vocation that to serve the fruit and vegetable's market that is hold in the same square every day. Finally, kiosks in Via Umberto, with those of Piazza Iolanda and Piazza Trento, serve the arteries with elegant stores.

good of which the prices have been recorded are: "Mandarin and Lemon" (ML), " Almond Milk " (LM) and the folk drink "Soda, Lemon and Salt" (SL).

Data are presented in the following Table 1.

TABLE 1 - PRICES PRACTISED FROM DRINK KIOSKS IN THE CITY OF CATANIA.

Via Umberto-axis				
LOCATION	FIRM	DRINK *Mandarin and Lemon*	*Soda Lemon and Salt*	Almond Milk
Piazza Iolanda	Giammona	0,90	0,70	1,20
Piazza Umberto	Vezzosi	1,00	0,75	1,20
	Giammona	1,00	0,75	1,50
Piazza Trento	Sava	0,90	0,70	1,30
Via Etnea-axis				
LOCATION	FIRM	DRINK *Mandarin and Lemon*	*Soda Lemon and Salt*	Almond Milk
Piazza Borsa	Cremino	1,00	0,80	1,35
Piazza Carlo Alberto	Guarrera	0,90	0,70	1,30
	Tappeti	0,80	0,65	1,00
Piazza S.Spirito	Costa	0,90	0,70	1,20
Via Santa Maddalena	S. Agata La Vetere	0,80	0,60	1,00

Note: prices are in euro, recorded in May 2005.

Focus on the competition which is realized when two firms are localised in the same place (notice that there are two cases, one in every of the axis), we can observe price differential to notice that price competition in Via Umberto (limited to only one product of three products here considered), is less intense than that along the Via Etnea-axis (see Table 2).

TABLE 2.- DIFFERENCE IN PRICES

Difference in prices	ML	SL	LM
Via Umberto	0	0	0,30/25%
Piazza Carlo Alberto	0,10%	0,05 8%	0,30/30%

Is it possible to find some relationship among the theory just exposed and the empirical evidence here reported? Could we reconcile the cohabitation into the centre connected to the more high-levels of price recorded in Via Umberto, with the situation in Piazza Carlo Alberto, in which the central location is associated with the lowest level of recorded?

This two situations seem in contradiction each other, but both the situations can be explained inside the illustrated theoretical framework. Indeed, central location has a polarizing effect on prices that became higher if a collusive agreement is done, as argued by Jehiel (1992), or equal to marginal costs because of a Bertrand competition.

Hence, what happens in Via Umberto can be interpreted as the result of a collusive agreement that pushes upward the prices and leads both firms to the central location. The other situation can be interpreted instead, as the outcome of a market in which both stages are played and, since it is evident that in the real world location choice is more binding than that on price, competitive tensions unload on this last stage.

What makes possible collusion along Via Umberto is still an open matter. Obviously, it depends on behavioural parameters that cannot be inserted into the model; nevertheless, hypothesizing along Via Umberto a triangular distribution of the type of that introduced in this article, its "natural" push toward the centre deriving from this distribution form is able "to facilitate" the collusion on price. While the situation into Piazza Carlo Alberto, is very similar to the hypothesis of uniform distribution as nearly "uniform" is the position of the stands there located, therefore with smaller incentive to collude.

In other words and on extreme synthesis, the analyzed empirical experience, related to the drink kiosks, shows that consumers' distribution with a central peak doesn't imply the adoption of minimum differentiation strategy. But, it can reveal a meaningful incentive to the adoption of collusive behaviours on prices.

5. CONCLUSIONS

In this paper I have analysed the effects of the consumers' concentration towards the middle of the space of product characteristics, in a model of horizontal differentiation with quadratic transportation costs.

From the vast available literature it is well known the importance of transport cost function on the market equilibrium. This paper, instead, pointed to the effects of the consumer's distribution, considering a distribution function nearer to the real world than uniform distribution which is generally considered by the available models. Indeed, usually in the real world, "extreme positions" have a smaller relative weight.

Hypothesizing a consumers' distribution not already uniform, but with a central peak, the sturdiness of the minimum differentiation principle was verified in a different way. I shown, in particular, that even in the case of (symmetrical) triangular distribution with central peak, if firms compete in both two stage price-location game, they will not prefer to localize on the centre of the market; they will maintain a location next to the market's edges. Firms' optimal behaviour is to push the indifferent consumer to the centre of the unitary segment.

In the final part of this paper I have brought shortly some data of an empirical analysis concerning to drink kiosks in the city of Catania. This analysis has moved by the purpose to analyze in an "original" way the existing relationship between location and practiced price.

The economic interpretation of the gotten results is based, not only on models of differentiation and competition, but also on the possibility that the duopolist conclude collusive agreement on price.

Observed behaviours *central location - elevated prices* and *central location – low prices* can be explained, in particular, by observing the polarizing (to the rise or to the rebate) effect that the central location has on prices.

A limit-case should happen if firms, localised in the centre, fix prices so high that consumers located on the edge decide to do not make any purchase and so the size of the market will decrease. Indeed, if consumers' distribution is triangular, market's edge will be relatively empty and firms may choose this option without worrying about this market size decreasing.

In this context the triangular distribution can be view as a possible incentive to the collusion, but not sufficient to justify, from itself, firms' permanence into the centre.

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